Exercise 53

For the following exercises, sketch a graph of the quadratic function and give the vertex, axis of symmetry, and intercepts.

$$f(x) = x^2 - 2x$$

Solution

In order to more easily graph the quadratic function, write it in vertex form by completing the square. The following algebraic identity is necessary.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -2$, which means $B = -1$ and $B^2 = 1$. Add and subtract 1 from the right side and use the identity.

$$\begin{aligned} f(x) &= (x^2 - 2x + 1) - 1 \\ &= (x + (-1))^2 - 1 \\ &= (x - 1)^2 - 1 \end{aligned}$$

Therefore, the vertex is $(1, -1)$, and the axis of symmetry is $x = 1$. To determine the y-intercept, set $x = 0$.

$$f(0) = (0 - 1)^2 - 1 = (1) - 1 = 0$$

Therefore, the y-intercept is $(0, 0)$. To get the x-intercept, set $y = 0$ and solve the equation for x.

$$\begin{aligned} 0 &= (x - 1)^2 - 1 \\ 1 &= (x - 1)^2 \end{aligned}$$

Take the square root of both sides.

$$\sqrt{1} = \sqrt{(x - 1)^2}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed around $x - 1$.

$$|x - 1| = 1$$

Remove the absolute value sign by placing $\pm$ on the opposite side.

$$x - 1 = \pm 1$$

Add 1 to both sides.

$$x = 1 \pm 1$$

This means $x = \{0, 2\}$, and the x-intercepts are $(0, 0)$ and $(2, 0)$.

A graph of the function is shown below.